

MODIFICATION OF THE ESAMS MODEL TO INCLUDE RWR FALSE ALARMS, SIGNAL DROPOUTS, & SENSITIVITY

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Background

ESAMS Version 2.6.3, Section 2, Aircraft Signal Generation, addresses penetrator characteristics by incorporating the use of aircraft signature and vulnerability data in the model. The aircraft signal generator models the return of voltages from the aircraft to the radar receiver. The model assumes all penetrator RWR signals are received from the radar transmitter in the generation of ECM parameter techniques to the radar receiver such as: frequency spectrum, radiated power, antenna pattern, and turn-on/turn-off times.

Introduction

In detecting threat radar signals via the penetrator aircraft's RWR, we need to be concerned about three crucial parameters: receiver false alarms and the ECM response to these false alarm signals, undetected threat signals (signal dropout), and receiver sensitivity. False alarms are those falsely detected signals in the absence of transmitted threat radar signals while signal dropouts are the absence of a detected signal in the presence of an actual transmitted threat radar signal. Receiver sensitivity defines the minimum signal strength a receiver can detect above the background noise density. These parameters are the most common anomalies in the reception of RF signals by the penetrator's RWR, hence EW simulation models such as ESAMS should be cognizant of these anomalies. Therefore these models should also incorporate a signal to noise power ratio, a false alarm probability, and receiver sensitivity variabilities into the model to simulate a more realistic engagement scenario. A mathematical analysis of these anomalies is presented in Appendix A.

Noise only - False alarms

Let's assume there is no input signal, only input noise is present at the receiver's envelope detector, see Figure 1. A false alarm will occur whenever the noise waveform amplitude exceeds the receiver system's noise threshold voltage.

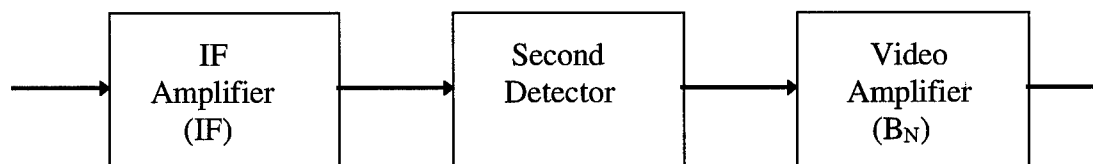


Figure 1: Envelope Detector

For a receiver system with similar IF (Intermediate Frequency) and video bandwidth¹, the voltage output from the envelope detector has a probability density function². The probability of a false alarm is identical to the probability that the detector output signal will exceed the threshold voltage (V_t). For narrowband, band limited noise, Papouli³ states that noise values will be independent at a rate equal to the reciprocal of the receiver

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system noise equivalent bandwidth (B_n). In order to maximize the receiver signal-to-noise power ratio (S/N), a bandpass filter is inserted prior to the envelope detector. For filter rolloffs much greater than two pole (40 dB/decade), the noise bandwidth is very close to the receiver system 3 dB bandwidth (to less than 1 dB error). This approximation improves rapidly with increasingly sharp filter rolloffs.

Average False Alarm Rate

Whenever the voltage envelope exceeds the receiver system threshold, a target detection is considered to have occurred, by definition⁴. Since the probability of a false alarm is the probability that noise will cross the threshold, the probability of a false alarm can be defined as P_{fa} .

$$P_{fa} = \exp(-V_t^2 / 2\psi_0) \text{ or } = \exp^{(-tnr)} \quad (1)$$

where: V_d = the receiver's detector output voltage
 ψ_0 = the variance (mean square) of the noise voltage
 A = the receiver input signal amplitude
 tnr = threshold to noise power ratio

Hence, the average time between false alarms (T_{fa}) can be evaluated for a particular receiver system noise equivalent bandwidth B_n as:

$$T_{fa} = 1 / P_{fa}B_n \quad (2)$$

If for example, the bandwidth of an IF amplifier were 1.0 Mhz and the average false-alarm time that could be tolerated were 15 minutes, the probability of a false alarm would be 1.11×10^{-9} . From equation 1, the threshold voltage necessary to achieve this false-alarm time is 6.44 times the rms value of the noise voltage. The point of interest is the relationship of average time between false alarms (T_{fa}) as a function of the threshold to noise ratio (TNR), as illustrated in Table 1.

TNR (dB)	T_{fa} (sec)
8	34.4 μ sec
9	176 μ sec
10	1.38 msec
11	18.3 msec
12	477 msec
13	28.9 sec
14	5.07×10^3 sec
15	3.38×10^6 sec

Table 1: Average Time Between False Alarms, Noise Equivalent Bandwidth = 16 Mhz

Table 1 confirms the well known behavior that average false alarm time is a very strong function of TNR; e.g. very small changes in TNR have enormous effects on T_{fa} . Thus a change in the threshold of only 1.0 dB changes the false-alarm time by many orders of magnitude. Such is the nature of Gaussian noise.

The false-alarm probabilities of present-day RWR receivers are usually quite small. The reason for this is that the false-alarm probability is the probability that a noise pulse will cross the threshold during an interval of time approximately equal to the reciprocal of the bandwidth. For a 1 Mhz receiver IF bandwidth, there are in the order of 10^6 noise pulses per second. Hence the false-alarm probability of any one pulse must be small ($< 10^{-6}$) if false-alarm times greater than 1 second are to be obtained.

The relationship between T_{fa} and B_N is interesting since they are inversely proportional to one another; e.g. the larger the equivalent noise bandwidth of the receiver, the shorter the probability of false-alarm time. As demonstrated in Table 1, a difference of 1 dB in the threshold to noise ratio between 13 to 15 dB results in a probability of false-alarm time from 29 seconds to 39 days. Applying this relationship to the ESAMS model, means the RWR threshold should be between 13 to 15 dB above the noise based on estimated mission specifications for the modeled penetrator aircraft. Assigning a value to the noise threshold (V_t), the receiver sensitivity, selectivity, probability of intercept, and bandwidth can then be determined. In practice, the threshold level should probably be adjusted above the value required by Equation 2, so that instabilities which lower the threshold slightly will not cause a flood of false alarms.

Signal Dropouts

If the receiver is desensitized to achieve a low false alarm rate (yield a higher tnr), the input signal may be of insufficient strength. Insufficient signal strength will produce a noise distribution from the envelope detector which is not far enough above the noise threshold (V_t) to prevent negative spikes from dropping below the threshold resulting in a loss of signal; i.e. signal dropout. The probability of false alarms, can be used to determine the probability of signal dropout (P_{do}). To obtain an average time between dropout events, the same analysis used to calculate the time between false alarms can be applied as:

$$T_{do} = 1 / P_{do} B_n \quad (3)$$

For simplification we can consider the signal dropouts as a series of discrete events, each lasting for a time $t = 1/B_n$, and happening at an average density of T_{do} . This relationship results in a Poisson density⁵ of dropout events, described by:

$$P_{(k,T)} = \left[(T/T_{do})^k \exp^{-T/T_{do}} \right] / K! \quad (4)$$

$P_{(k,T)}$ is the probability of (k) dropout events happening in an interval of (T) seconds. Since at any particular snr and tnr, the process is stationary, this model can be applied to individual pulses independently. Therefore the probability of losing a signal (pulse) completely, due to noise, can be found by setting $T = PW$ (pulse width), and letting $K =$ the number of contiguous events necessary to equal or exceed the time duration of the

input signal (pulse). The point of interest is the relationship of the probability of signal dropouts as the function of the threshold to noise ratio (TNR), as illustrated in Table 2.

TNR (dB)	P _{DO}
9	6.22×10^{-7}
10	14.9×10^{-6}
11	3.05×10^{-4}
12	4.52×10^{-3}
13	3.68×10^{-2}
14	0.137

Table 2: Probability of Signal Dropouts, Noise Equivalent Bandwidth = 16 Mhz

Two relationships are interesting from Table 2. First, the probability of a signal dropout is proportional to an increase in the threshold to noise ratio (TNR), and second, the probability of a signal dropout event increases with pulse width when the pulse width becomes equal to or greater than the reciprocal of the bandwidth; hence obliterating the pulse. In other words, as the threshold to noise ratio of the receiver increases, the probability of a signal not being detected by the receiver also increases. This relationship indicates that due to some conditions of background clutter (noise), threat signal detection anomalies may occur as a result of background noise.

Sensitivity

Receiver sensitivity defines the minimum signal strength a receiver can receive and still perform the task it is intended to do. Sensitivity is a power level, typically stated in dBm (usually a large negative number of dBm). There are three component parts of receiver sensitivity: the thermal noise level (called kTB), the receiver system noise figure and the signal-to-noise ratio required to adequately recover the desired information from the received signal.

The sensitivity of a receiver (or system) is the sum of the thermal noise plus the noise figure plus the signal-to-noise ratio. As an example, if the receiving system has an effective bandwidth of 16 MHz (kTB = -114 dBm/MHz), a noise figure of 10 dB, and a SNR of 15 dB, the sensitivity of the receiving system is:

$$\begin{aligned} \text{Sensitivity} &= \text{kTB} + \text{noise figure} + \text{SNR} \\ &= (-114 \text{ dBm} + 12 \text{ dB}) + 10 \text{ dB} + 15 \text{ dB} = -77 \text{ dBm} \end{aligned}$$

Since one of the components affecting receiver sensitivity is the signal to noise ratio (SNR) which is affected by the threshold to noise ratio (TNR), receiver sensitivity will determine the probability of false alarms and signal dropouts. Thus receiver sensitivity is a function of the receiver's ability to minimize both the signal false alarm rate and dropout rate.

Conclusion

The preceding discussion has presented the impact on receiver false alarms, signal dropouts, and sensitivity. If the threshold to noise ratio is too high, the receiver's sensitivity will be decreased resulting in its inability to detect low level signals. During DT&E and OT&E tests, the RF monitoring of threat radar reception and transmission as a function of time as well as monitoring aircraft receiver detected signals and ECM response as a function of time is monitored to alert the testers of any RF anomalies. Post data processing is performed and the data analyzed after each test sortie as well as before the conclusion of range tests to assure themselves that they have valid RF threat, RWR, and background data. The concern with receiver false alarms, signal dropouts and sensitivity suggests that they should be incorporated into the construct of the ESAMS simulation model. In reality, there exists a strong potential for the penetrator aircraft's RWR to detect false targets and impact an unintentional response from the ECM equipment or to cause signal dropouts which would not cause an ECM response to an actual threat signal. These anomalies are actually introduced into the receiver system by environmental multipath effects (reflections) from the terrain clutter. Since these actual anomalies due to the physics of the environment, we should incorporate the parameters which contribute to these anomalies into the simulation model..

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APPENDIX A

Noise only - False Alarms

Let's assume there is no input signal, only input noise is present at the receiver's envelope detector, see Figure 1. A false alarm will occur whenever the noise waveform amplitude exceeds the receiver system's noise threshold voltage.

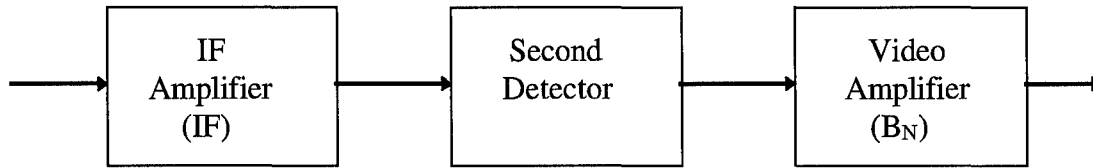


Figure 1: Envelope Detector

For a receiver system with similar IF (Intermediate Frequency) and video bandwidth¹, the voltage output from the envelope detector has a probability density function² of:

$$P(V_d) = V_d / \psi_0 \exp - (V_d^2 + A^2) / 2\psi_0 \quad (1)$$

where: V_d = the receiver's detector output voltage
 ψ_0 = the variance (mean square) of the noise voltage
 A = the receiver input signal amplitude

With no receiver input signal ($A = 0$), Equation one (1) reduces to Equation two (2):

$$P(V_d) |_{A=0} = V_d / \psi_0 \exp - V_d^2 / 2\psi_0 \quad (2)$$

which is a form of the Rayleigh density function. The probability of a false alarm is identical to the probability that the detector output signal will exceed the threshold voltage (V_t):

$$P_{fa} = \int_{V_t}^{\infty} P(V_d) |_{A=0} dV_d = \int_{V_t}^{\infty} V_d / \psi_0 \exp - V_d^2 / 2\psi_0 dV_d = \exp - V_t^2 / 2\psi_0 \quad (3)$$

To convert Equation three (3) to a "rate" of false alarms, from a probability that one will occur, requires knowledge of the average rate of independent noise events in the receiver system. For narrowband, band limited noise, Papouli³ states that noise values will be independent at a rate equal to the reciprocal of the receiver system noise equivalent bandwidth (B_n). In order to maximize the receiver signal - to - noise (S/N), a bandpass filter is inserted prior to the envelope detector. For filter rolloffs much greater than two pole (40 dB / decade), the noise bandwidth is very close to the receiver system 3 dB bandwidth (to less than 1 dB error).

To evaluate Equation three (3), we can define a sine wave rms equivalent threshold to noise power ratio (tnr) as:

$$\text{tnr} = V_t^2 / 2\psi_0 \quad (4)$$

Equation four (4) has the same form as the signal to noise (snr) ratio:

$$\text{snr} = (S/N) = A^2 / 2\psi_0 \quad (5)$$

Note that a signal to threshold ratio (str) can be defined as the difference between the signal and threshold voltages and scaled by the noise power, which is the square of the ratios, not the difference between Equations five (5) and four (4), namely:

$$\text{str} = (A - V_t)^2 / 2\psi_0 \quad \neq \quad \text{snr} - \text{tnr} = (A^2 - V_t^2) / 2\psi_0$$

Therefore the problem of noise dropouts during the presence of a signal cannot be modeled as a noise process with probabilities defined by Equation three (3) by substituting $A - V_t$ for V_t . The actual probability density function must be evaluated, and is discussed in the section on signal dropouts.

Average False Alarm Rate

Since the probability of a false alarm is the probability that noise will cross the threshold, the probability of a false alarm can be defined as P_{fa} :

$$P_{fa} = \exp(-V_t^2 / 2\psi_0) \quad \text{or} \quad = \exp^{(-\text{tnr})} \quad (6)$$

Hence, the average time between false alarms (T_{fa}) can be evaluated for various tnr values, at a particular receiver system noise equivalent bandwidth B_n as :

$$T_{fa} = 1 / P_{fa} B_n \quad (7)$$

The point of interest is the relationship of average time between false alarms (T_{fa}) as a function of the threshold to noise power ratio ($\text{TNR} = 10 \log \text{tnr}$), see Table 1.

TNR (dB)	T_{fa} (sec)
8	34.4 μsec
9	176 μsec
10	1.38 msec
11	18.3 msec
12	477 msec
13	28.9 sec
14	5.07×10^3 sec
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Table 1: Average Time Between False Alarms, Noise Equivalent Bandwidth = 16 Mhz

The relationship between T_{fa} and B_n is interesting since they are inversely proportional to one another; e.g. the larger the equivalent noise bandwidth of the receiver, the shorter the probability of false-alarm time. As demonstrated in Table 1, a difference of 1 dB in the threshold to noise ratio between 13 to 15 dB results in a probability of false-alarm time from 29 seconds to 39 days. This means the RWR threshold should be between 13 to 15

dB above the noise based on estimated aircraft missions. From the value of the noise threshold (V_t), the receiver sensitivity, selectivity, probability of intercept, and bandwidth can then be determined. In practice, the threshold level should probably be adjusted above that required by Equation 7, so that instabilities which lower the threshold slightly will not cause a flood of false alarms.

Signal Dropouts

If the receiver is desensitized to achieve a low false alarm rate (yield a higher tnr), the input signal may be of insufficient strength. Insufficient signal strength will produce a noise distribution from the envelope detector which is not far enough above the noise threshold (V_t) to prevent negative spikes from dropping below the threshold resulting in a loss of signal; i.e. signal dropout.

The probability of signal dropout must be calculated from the probability density function, Equation one (1), where the desired result is the probability that the envelope detector output V_d is less than V_t ($V_d < V_t$). Also every measurement of the detector output must yield some value, the integral of the probability density over all values of voltage must be equal to unity:

$$\int_{-\infty}^{\infty} P(v_d) dv_d = 1 \quad (8)$$

Therefore:

$$P_{do} = P(V_d < V_t) = \int_{-\infty}^{V_t} P(v_d) dv_d = 1 - \int_{V_t}^{\infty} v_d / \psi_0 \exp - [(v_d^2 + A^2) / 2\psi_0] dv_d \quad (9)$$

Using the series expansion for $I_0(X)$ of:

$$I_0(X) = 1 + (x^2 / 2^2) + (x^4 / 2^2 \cdot 4^2) + \dots = \sum_{n=0}^{\infty} x^{2n} / 2^{2n} (n!)^2$$

The integral in Equation ten (9) evaluates to the infinite series:

$$P_{do} = 1 - \sum_{n=0}^{\infty} \{ (-1)^n \text{snr}^n / (n!)^2 \exp^{-(\text{snr} + \text{tnr})} [\sum_{k=0}^{\infty} (-1)^k n! / (n-k)! (-\text{tnr})^{n-k}] \} \quad (10)$$

This probability calculation (P_{do}) is valid for all values of snr, provided enough terms are used to allow convergence to required tolerances.

Analogous to Equation three (3), the probability of false alarms, Equation eleven (10) will determine the probability of signal dropout (P_{do}). To obtain an average time between

dropout events, the same analysis used to calculate the time between false alarms, Equation seven (7), can be applied as:

$$T_{do} = 1 / P_{do} B_n \quad (11)$$

For simplification we can consider the signal dropouts as a series of discrete events, each lasting for a time $t = 1/B_n$, and happening at an average density of T_{do} . This relationship results in a Poisson density⁵ of dropout events, described by:

$$P_{(k,T)} = [(T/T_{do})^k \exp^{-T/T_{do}}] / K! \quad (12)$$

$P_{(k,T)}$ is the probability of (k) dropout events happening in an interval of (T) seconds. Since at any particular snr and tnr, the process is stationary, this model can be applied to individual pulses independently. Therefore, the probability of losing a signal (pulse) completely, due to noise, can be found from Equation thirteen (13) by setting $T = PW$ (pulse width), and letting K = the number of contiguous events necessary to equal or exceed the time duration of the input signal (pulse). The point of interest is the relationship of the probability of signal dropouts as the function of the threshold to noise ratio (TNR), as illustrated in Table 2.

TNR (dB)	P _{do}
9	6.22×10^{-7}
10	14.9×10^{-6}
11	3.05×10^{-4}
12	4.52×10^{-3}
13	3.68×10^{-2}
14	0.137

Table 2: Probability of Signal Dropouts, Noise Equivalent Bandwidth = 16 Mhz

Two relationships are evident as revealed by Equation 12 and Table 2. First, the probability of a signal dropout is proportional to an increase in the threshold to noise ratio (TNR), and second, probability of a signal dropout event increases with pulse width when the pulse width becomes equal to or greater than the reciprocal of the bandwidth; hence obliterating the pulse. In other words, as the threshold to noise ratio of the receiver increases, the probability of a signal not being detected by the receiver also increases. This relationship indicates that due to some conditions of background clutter (noise), threat signal detection anomalies may occur as a result of background noise.

Sensitivity

Receiver sensitivity defines the minimum signal strength a receiver can receive and still perform the task it is intended to do. Sensitivity is a power level, typically stated in dBm (usually a large negative number of dBm). There are three component parts of receiver sensitivity: the thermal noise level (called kTB), the receiver system noise figure and the

signal-to-noise ratio required to adequately recover the desired information from the received signal.

Thermal Noise (kTB)

kTB is a product of three values which defines the receiver thermal noise level in an ideal receiver.

- k is Boltzman's constant (1.38×10^{-23} Joule/°K)
- T is the operating temperature in degrees Kelvin
- B is the effective receiver bandwidth in Hertz.

When the operating temperature is set at 290° K (a standard condition used to represent "room" temperature but is actually a cool 17° C or 63° F) and the receiver bandwidth is set at 1 Mhz, the converted value of kTB in decibels is:

$$\boxed{kTB = -114 \text{ dBm/MHz}}$$

Noise Figure

The ratio of the noise injected into the input of an ideal, noiseless receiver (or receiver system) to produce the noise that is actually present at its output is called noise figure (noise/kTB). The noise figure of a receiver is the amount of thermal noise the receiver adds to a received signal, referenced to the receiver input. As an example, the noise figure of an amplifier is the thermal noise source having an equivalent generator resistance equal to that specified for use with the receiver.

$$NF = 10 \log (N_2/N_1) \quad \text{where } N_2 \text{ is the receiver input noise with its amplifier ON and } N_1 \text{ is the receiver input noise with its amplifier OFF.}$$

Signal-to-Noise Ratio

The signal-to-noise power ratio (SNR) of a receiver is that amount of signal which is 1 dB or greater than the noise at the input of a noiseless receiver.

Therefore, the sensitivity of a receiver (or system) is the sum of the thermal noise plus the noise figure plus the signal-to-noise ratio. As an example, if a receiving system has an effective bandwidth of 16 MHz ($kTB = -114 \text{ dBm/MHz}$), a noise figure of 10 dB, and a SNR of 15 dB, the sensitivity of the receiving system is:

$$\begin{aligned} \text{Sensitivity} &= kTB + \text{noise figure} + \text{SNR} \\ &= (-114 \text{ dBm} + 12 \text{ dB}) + 10 \text{ dB} + 15 \text{ dB} = -77 \text{ dBm} \end{aligned}$$

Thermal Noise Derivation

Thermal noise is the noise generated by electron flow through a resistor or in active devices across their junctions. The phenomenon is based on three parameters: Boltzman's constant (1.38×10^{-23} joules/ $^{\circ}$ K), the operating temperature in degrees Kelvin, and the effective bandwidth in Hertz.

When the operating temperature is set at 290° K (a standard condition used to represent "room" temperature), and the receiver bandwidth is set at 1 Mhz, the converted value of kTB in decibels (in dBm) is:

$$\begin{aligned} \text{kTB} &= (1.38 \times 10^{-23}) (2.90 \times 10^2) (1 \times 10^6) = 4.002 \times 10^{-15} \\ &= 10 \log (4.002 \times 10^{-15} \times 10^3) = 10 \log 4.002 \times 10^{-12} \\ &= 10 \times -11.3977 \\ &= -113.977 \end{aligned}$$

kTB \approx -114 dBm / MHz
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NOTE: 1. The 10^3 factor is for converting dB to dBm
2. Joules = Watt-seconds
3. Frequency = 1/Time

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